

1.

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . When P is hanging vertically below O , it is given a horizontal speed u . In the subsequent motion, P moves in a complete circle. When OP makes an angle θ with the downward vertical, the tension in the string is T . Show that

$$T = \frac{mu^2}{a} + mg(3 \cos \theta - 2). \quad [5]$$

Given that the ratio of the maximum value of T to the minimum value of T is 3 : 1, find u in terms of a and g . [4]

Assuming this value of u , find the value of $\cos \theta$ when the tension is half of its maximum value. [3]

2.

AB is a diameter of a uniform circular disc D of mass $9m$, radius $3a$ and centre O . A lamina is formed by removing a circular disc, with centre O and radius a , from D . Show that the moment of inertia of the lamina, about a fixed horizontal axis l through A and perpendicular to the plane of the lamina, is $112ma^2$. [5]

A particle of mass $3m$ is now attached to the lamina at B . The system is free to rotate about the axis l . The system is held with B vertically above A and is then slightly displaced and released from rest. The greatest speed of B in the subsequent motion is $k\sqrt{ga}$. Find the value of k , correct to 3 significant figures. [7]

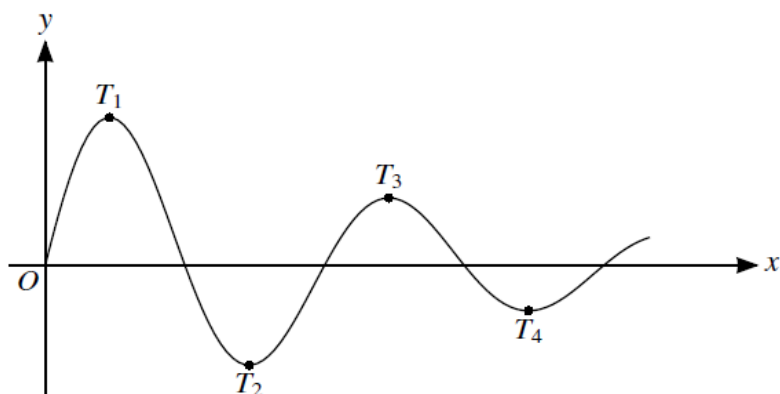
3.

A car of mass 1200 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 20 kW. The resistance to the car's motion is constant and equal to 500 N. The car passes through the points A and B with speeds 10 m s^{-1} and 25 m s^{-1} respectively. The car takes 30.5 s to travel from A to B .

(i) Find the acceleration of the car at A . [4]

(ii) By considering work and energy, find the distance AB . [8]

4.



The diagram shows the curve $y = 10e^{-\frac{1}{2}x} \sin 4x$ for $x \geq 0$. The stationary points are labelled T_1 , T_2 , T_3 , ... as shown.

(i) Find the x -coordinates of T_1 and T_2 , giving each x -coordinate correct to 3 decimal places. [6]

(ii) It is given that the x -coordinate of T_n is greater than 25. Find the least possible value of n . [4]

5.

Let $I_n = \int_0^1 \frac{1}{(1+x^4)^n} dx$. By considering $\frac{d}{dx} \left(\frac{x}{(1+x^4)^n} \right)$, show that

$$4nI_{n+1} = \frac{1}{2^n} + (4n-1)I_n. \quad [4]$$

Given that $I_1 = 0.866\,97$, correct to 5 decimal places, find I_3 . [4]

6.

Show that if λ is an eigenvalue of the square matrix \mathbf{A} with \mathbf{e} as a corresponding eigenvector, and μ is an eigenvalue of the square matrix \mathbf{B} for which \mathbf{e} is also a corresponding eigenvector, then $\lambda + \mu$ is an eigenvalue of the matrix $\mathbf{A} + \mathbf{B}$ with \mathbf{e} as a corresponding eigenvector. [2]

The matrix

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 0 \\ -4 & -6 & -6 \\ 5 & 11 & 10 \end{pmatrix}$$

has $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ as an eigenvector. Find the corresponding eigenvalue. [1]

The other two eigenvalues of \mathbf{A} are 1 and 2, with corresponding eigenvectors $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ respectively. The matrix \mathbf{B} has eigenvalues 2, 3, 1 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ respectively. Find a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $(\mathbf{A} + \mathbf{B})^4 = \mathbf{PDP}^{-1}$. [3]

[You are not required to evaluate \mathbf{P}^{-1} .]

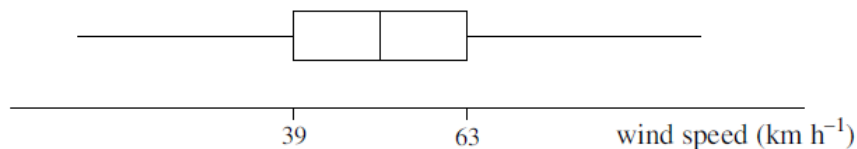
7.

Two planes have equations $x + 2y - 2z = 2$ and $2x - 3y + 6z = 3$. The planes intersect in the straight line l .

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line l . [6]

8.



Measurements of wind speed on a certain island were taken over a period of one year. A box-and-whisker plot of the data obtained is displayed above, and the values of the quartiles are as shown. It is suggested that wind speed can be modelled approximately by a normal distribution with mean μ km h⁻¹ and standard deviation σ km h⁻¹.

- (i) Estimate the value of μ . [1]
- (ii) Estimate the value of σ . [3]

9.

In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

- (i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]
- (ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]
- (iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

10.

For each month of a certain year, a weather station recorded the average rainfall per day, x mm, and the average amount of sunshine per day, y hours. The results are summarised below.

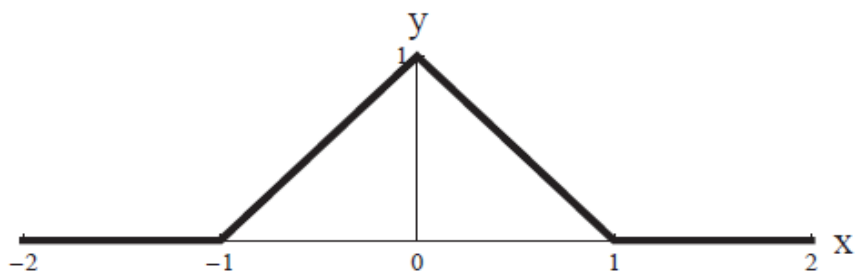
$$n = 12, \quad \Sigma x = 24.29, \quad \Sigma x^2 = 50.146, \quad \Sigma y = 45.8, \quad \Sigma y^2 = 211.16, \quad \Sigma xy = 88.415.$$

- (i) Find the mean values, \bar{x} and \bar{y} . [1]
- (ii) Calculate the gradient of the line of regression of y on x . [2]
- (iii) Use the answers to parts (i) and (ii) to obtain the equation of the line of regression of y on x . [2]
- (iv) Find the product moment correlation coefficient and comment, in context, on its value. [4]
- (v) Stating your hypotheses, test at the 1% level of significance whether there is negative correlation between average rainfall per day and average amount of sunshine per day. [4]

11.

[5]

G. A graph of the function $y = f(x)$ is sketched on the axes below:



The value of $\int_{-1}^1 f(x^2 - 1) dx$ equals

12. Choose **ONLY ONE** from next two questions to answer.

[10]

12.1

Two particles of masses m and M , with $M > m$, lie in a smooth circular groove on a horizontal plane. The coefficient of restitution between the particles is e . The particles are initially projected round the groove with the same speed u but in opposite directions. Find the speeds of the particles after they collide for the first time and show that they will both change direction if $2em > M - m$.

After a further $2n$ collisions, the speed of the particle of mass m is v and the speed of the particle of mass M is V . Given that at each collision both particles change their directions of motion, explain why

$$mv - MV = u(M - m),$$

and find v and V in terms of m , M , e , u and n .

12.2

The number of texts that George receives on his mobile phone can be modelled by a Poisson random variable with mean λ texts per hour. Given that the probability George waits between 1 and 2 hours in the morning before he receives his first text is p , show that

$$pe^{2\lambda} - e^\lambda + 1 = 0.$$

Given that $4p < 1$, show that there are two positive values of λ that satisfy this equation.

The number of texts that Mildred receives on each of her two mobile phones can be modelled by independent Poisson random variables with different means λ_1 and λ_2 texts per hour. Given that, for each phone, the probability that Mildred waits between 1 and 2 hours in the morning before she receives her first text is also p , find an expression for $\lambda_1 + \lambda_2$ in terms of p .

Find the probability, in terms of p , that she waits between 1 and 2 hours in the morning to receive her first text.